

4-6 Cramer's Rule

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- used determinants to solve systems of equations

Use elimination to solve for x:

$$\begin{array}{rcl} d(ax + by = e) & \times & 3y \\ -b(cx + dy = f) & \times & -3 \end{array}$$

$$\begin{array}{r} adx + bdy = de \\ -bcx - bdy = -bf \\ \hline (ad - bc)x = de - bf \\ x = \frac{de - bf}{ad - bc} \end{array}$$

$$\begin{array}{c} \text{denom.} \\ \begin{vmatrix} a & b \\ c & d \end{vmatrix} \end{array} \quad \begin{array}{c} \text{num.} \\ \begin{vmatrix} e & b \\ f & d \end{vmatrix} \end{array}$$

$$x = \frac{de - bf}{ad - bc}$$

$$\begin{array}{l} ax + by = e \\ cx + dy = f \end{array}$$

Denominator is:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Write a determinant matrix for the numerator:

Cramer's Rule:

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$$

Where D_x replaces the coefficients of x with the constants and D_y replaces the coefficients of y with the constants.

$$\begin{array}{l} ax + by = e \\ cx + dy = f \end{array}$$

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix} \quad D_y = \begin{vmatrix} a & e \\ c & f \end{vmatrix}$$

ex:

$$x - 3y = 5$$

$$2x + 9y = -10$$

$$D = \begin{vmatrix} 1 & -3 \\ 2 & 9 \end{vmatrix} = 15$$

$$D_x = \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = 15$$

$$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -10 \end{vmatrix} = -20$$

$$(1, -\frac{4}{3})$$

$$x = \frac{D_x}{D} = \frac{15}{15}$$

$$x = 1$$

$$y = \frac{D_y}{D} = \frac{-20}{15}$$

$$y = -\frac{4}{3}$$

If $D = 0$

then, either

 ∞ # of solutions

OR

 \emptyset

ex:

$$2x - y = 3$$

$$6x - 3y = -9$$

$$2x - y = 3$$

$$2x - y = -3$$

$$D = \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = 0$$

 \emptyset

3 Variables

Cramer's Rule:

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D} \quad z = \frac{D_z}{D}$$

ex:

$$4x + y - z = -2$$

$$x + 3y - 4z = 1$$

$$2x - y + 3z = 4$$

$$D = \begin{vmatrix} 4 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 3 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & -1 & 3 \end{vmatrix} = -16$$

$$D_y = \begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -4 \\ 2 & 4 & 3 \end{vmatrix} = 96$$

$$(-1, 6, 4)$$

$$\frac{-16}{16} \quad \frac{96}{16} \quad \frac{64}{16}$$

ex:

$$3x - 2y + 2z = 1$$

$$2x + 5y - 5z = 7$$

$$4x - 3y + z = -3$$

$$D = -38$$

$$D_x = -38$$

$$D_y = -114$$

$$D_z = -76$$

$$(1, 3, 2)$$

HW

p 192-193

12, 13, ~~26~~ 28, 30